Waves in Linear Optical Media

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Outline

- Plane waves in free space. Polarization.
- Plane waves in linear lossy media.
- Dispersion relations for linear waves in dispersionless, homogeneous media.
- Waves in anisotropic media: Faraday rotation, uniaxial crystals.
- Waves in piecewise-continuous media: Fresnel theory.
- Brewster and surface plasmon-polariton modes.

Light propagation in free space

Maxwell's equations (ME) in free space:

$$\nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t},$$

$$\nabla \times \mathbf{H} = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t},$$

$$\nabla \cdot \mathbf{E} = 0,$$

$$\nabla \cdot \mathbf{H} = 0.$$

Describe any classical optics phenomenon in free space!

Eliminating the magnetic field:

$$\nabla \times (\nabla \times \mathbf{E}) = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = -\mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}.$$

recalling that $\mu_0 \varepsilon_0 = 1/c^2$,

$$\boxed{\nabla^2 \mathbf{E} - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0.} \qquad (1)$$

- Wave equation in free space.

There exist plane wave solutions

Plane wave solutions:

$$\mathbf{E} = \mathbf{E}_0 e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)},$$

$$\mathbf{H} = \mathbf{H}_0 e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)},$$

with the dispersion relation:

$$\boxed{k^2 = \omega^2 / c^2}$$

as well as the relations

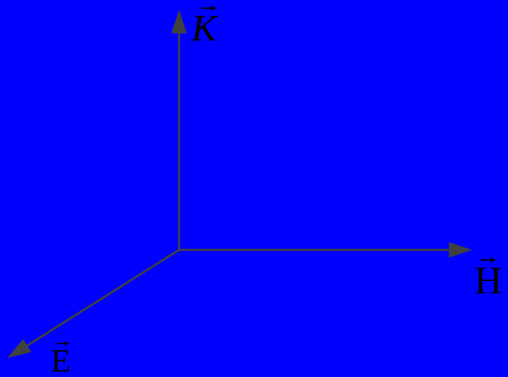
$$\boxed{\mathbf{H}_0 = \frac{(\hat{\mathbf{e}}_k \times \mathbf{E}_0)}{\eta_0}}$$

$$\boxed{\mathbf{E}_0 = -\eta_0(\hat{\mathbf{e}}_k \times \mathbf{H}_0)}.$$

where $\eta_0 = \sqrt{\mu_0/\varepsilon_0}$ (free space impedance).

The transversality conditions:

$$\hat{\mathbf{e}}_k \cdot \mathbf{E}_0 = 0, \qquad \hat{\mathbf{e}}_k \cdot \mathbf{H}_0 = 0.$$



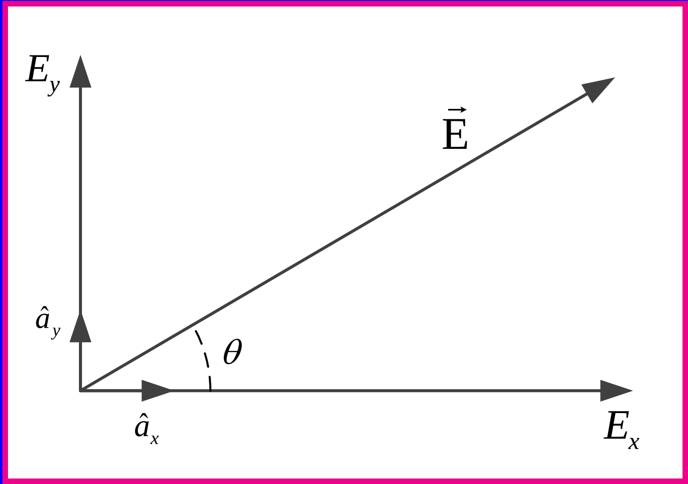
$$\mathbf{E} = (E_x\hat{\mathbf{e}}_x + E_y\hat{\mathbf{e}}_y)e^{i(kz-\omega t)},$$

$$E_x = |E_x|e^{i\phi_x}, \qquad\qquad E_y = |E_y|e^{i\phi_y}.$$

- $\phi_x = \phi_y$, linear polarization,
- $|E_x| = |E_y|$, $\phi_x - \phi_y = \pi/2$, circular polarization
- Otherwise, elliptical polarization.

The elliptic polarization is the most general case,

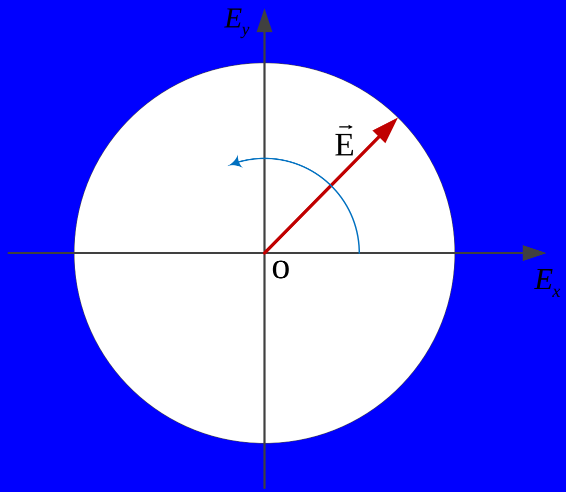
$$|E_x| \neq |E_y| \qquad \text{and} \qquad \phi_x \neq \phi_y.$$



Linear polarization,

$$E = \sqrt{E_x^2 + E_y^2}, \qquad \theta = \tan^{-1}(E_y/E_x).$$

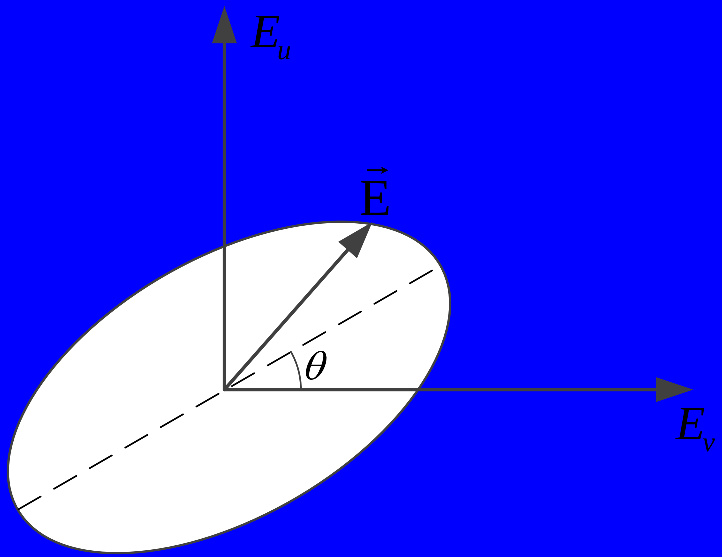
Circular polarization,



$$\hat{\mathbf{e}}_\pm = \frac{\hat{\mathbf{e}}_x \pm i\hat{\mathbf{e}}_y}{\sqrt{2}},$$

$$\hat{\mathbf{e}}_\pm^* \cdot \hat{\mathbf{e}}_\mp = 0, \qquad \hat{\mathbf{e}}_\pm^* \cdot \hat{\mathbf{e}}_\pm = 1.$$

Elliptical polarization,



$$\mathbf{E} = (E_+\hat{\mathbf{e}}_+ + E_-\hat{\mathbf{e}}_-)e^{i(kz-\omega t)}, \qquad E_-/E_+ = re^{i\alpha}.$$

Light propagation in linear isotropic lossy media

Constitutive relations

$$\mathscr{D} = \varepsilon_0 \varepsilon(\omega) \mathscr{E}, \qquad \mathscr{B} = \mu_0 \mathscr{H},$$

and

$$\varepsilon(\omega) = \varepsilon'(\omega) + i\varepsilon''(\omega),$$

Maxwell's equations

$$\mathbf{k} \cdot \mathscr{E} = 0,$$

$$\mathbf{k} \cdot \mathscr{H} = 0,$$

$$\mathbf{k} \times \mathscr{E} = \mu_0 \omega \mathscr{H}$$

$$\mathbf{k} \times \mathscr{H} = -\varepsilon_0 \varepsilon(\omega) \omega \mathscr{E}.$$

Seeking inhomogeneous plane wave solutions

$$\mathbf{E}(z,t) = \mathrm{Re}\{\mathscr{E} e^{i(kz - i\omega t)}\}$$

$$\mathbf{H}(z,t) = \mathrm{Re}\{\mathscr{H} e^{i(kz - i\omega t)}\}$$

Dispersion relation

$$k = \beta_\pm + i\alpha_\pm/2.$$

where

$$\beta_\pm = \pm\frac{\omega}{2c}\sqrt{\sqrt{\varepsilon'^2+\varepsilon''^2}+\varepsilon'}$$

and

$$\alpha_\pm = \pm\frac{\omega}{c}\sqrt{\sqrt{\varepsilon'^2+\varepsilon''^2}-\varepsilon'}$$

The electric and magnetic field amplitudes:

$$\mathscr{E} = -\eta(\hat{\mathbf{e}}_z \times \mathscr{H}),$$

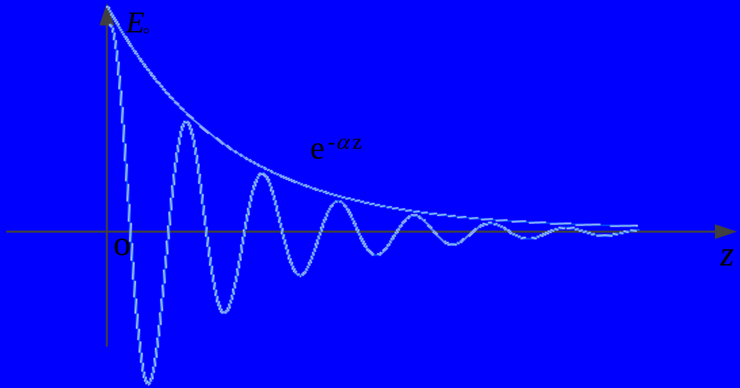
and

$$\mathscr{H} = \frac{(\hat{\mathbf{e}}_z \times \mathscr{E})}{\eta},$$

η being a complex impedance of the medium

$$\eta = \frac{\eta_0}{\sqrt{\varepsilon(\omega)}}$$

$$|\eta| = \frac{\eta_0}{(\varepsilon'^2 + \varepsilon''^2)^{1/4}}, \qquad \tan\theta_\eta = -\varepsilon''/\varepsilon'$$



Linearly polarized plane wave

$$\mathbf{E}(z,t) = \hat{\mathbf{e}}_x \mathscr{E} e^{-\alpha z} \cos(\beta z - \omega t)$$

and

$$\mathbf{H}(z,t) = \hat{\mathbf{e}}_y \frac{\mathscr{E}}{|\eta|} e^{-\alpha z} \cos(\beta z - \omega t - \theta_\eta).$$

Lossless dielectrics, $\varepsilon'' = 0$

$$\mathbf{E}(z,t) = \hat{\mathbf{e}}_x \mathscr{E} \cos(\beta z - \omega t),$$

$$\mathbf{H}(z,t) = \hat{\mathbf{e}}_y \frac{\mathscr{E}}{|\eta|} \cos(\beta z - \omega t).$$

Homogeneous plane wave with parameters

$$\lambda = 2\pi/\beta,$$

$$v_p = \omega/\beta.$$

Light propagation in linear anisotropic media

MEs in source-free nonmagnetic media

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t},$$

$$\nabla \cdot \mathbf{D} = 0,$$

$$\nabla \cdot \mathbf{B} = 0.$$

$$\mathbf{B} = \mu_0 \mathbf{H}$$

Eliminating magnetic field from MEs:

$$\nabla(\nabla\cdot\mathbf{E}) - \nabla^2\mathbf{E} = -\mu_0\frac{\partial^2\mathbf{D}}{\partial t^2}$$

and

$$\nabla\cdot\mathbf{D} = 0,$$

The electric flux density in such media:

$$D_i = \sum_{j=1}^{3}\varepsilon_{ij}E_j,$$

Plane-wave solution for the field:

$$E_i = \frac{1}{2}\mathscr{E}_i e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} + c.c.,.$$

and the flux density:

$$D_i = \frac{1}{2}\sum_j \varepsilon_{ij}\mathscr{E}_j e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} + c.c..$$

Modes supported by the medium:

$$\sum_j \left(k_i k_j - k^2 \delta_{ij} + \frac{\omega^2}{c^2}\varepsilon_{ij} \right) \mathscr{E}_j = 0,$$

$$\begin{pmatrix} k_y^2 + k_z^2 - \frac{\omega^2}{c^2}\varepsilon_{xx} & -k_x k_y - \frac{\omega^2}{c^2}\varepsilon_{xy} & -k_x k_z - \frac{\omega^2}{c^2}\varepsilon_{xz} \\ -k_x k_y - \frac{\omega^2}{c^2}\varepsilon_{yx} & k_x^2 + k_z^2 - \frac{\omega^2}{c^2}\varepsilon_{yy} & -k_y k_z - \frac{\omega^2}{c^2}\varepsilon_{yz} \\ -k_x k_z - \frac{\omega^2}{c^2}\varepsilon_{zx} & -k_y k_z - \frac{\omega^2}{c^2}\varepsilon_{zy} & k_x^2 + k_y^2 - \frac{\omega^2}{c^2}\varepsilon_{zz} \end{pmatrix}$$

$$\times \begin{pmatrix} \mathcal{E}_x \\ \mathcal{E}_y \\ \mathcal{E}_z \end{pmatrix} = 0 \qquad (2)$$

or

$$\sum_j \left[k^2 \left(\delta_{ij} - \frac{k_i k_j}{k^2} \right) - \frac{\omega^2}{c^2} \varepsilon_{ij} \right] \mathcal{E}_j = 0,$$

plus the transversality condition:

$$\sum_{i,j} k_i \varepsilon_{ij} \mathscr{E}_j = 0.$$

Dispersion relations:

$$\text{Det}\left(k_i k_j - k^2 \delta_{ij} + \frac{\omega^2}{c^2} \varepsilon_{ij} \right) = 0.$$

or

$$\begin{vmatrix} k_y^2 + k_z^2 - \frac{\omega^2}{c^2}\varepsilon_{xx} & -k_x k_y - \frac{\omega^2}{c^2}\varepsilon_{xy} & -k_x k_z - \frac{\omega^2}{c^2}\varepsilon_{xz} \\ -k_x k_y - \frac{\omega^2}{c^2}\varepsilon_{yx} & k_x^2 + k_z^2 - \frac{\omega^2}{c^2}\varepsilon_{yy} & -k_y k_z - \frac{\omega^2}{c^2}\varepsilon_{yz} \\ -k_x k_z - \frac{\omega^2}{c^2}\varepsilon_{zx} & -k_y k_z - \frac{\omega^2}{c^2}\varepsilon_{zy} & k_x^2 + k_y^2 - \frac{\omega^2}{c^2}\varepsilon_{zz} \end{vmatrix} = 0$$

Faraday rotation in external magnetic field

The dielectric tensor (phenomenological)

$$\varepsilon_{ij}(\omega) = \varepsilon(\omega)\delta_{ij} + ig(\omega)e_{ijl}B_{0l},$$

$$gB_0 \ll \varepsilon.$$

$\mathbf{B}_0 \Leftrightarrow$ weak magnetic field.

$$e_{ijk} = \begin{cases} 1 & \text{even permutation of ijk;} \\ -1 & \text{odd permutation of ijk} \end{cases}$$

Assume $\mathbf{k} = (0, 0, k)$; Dispersion relation:

$$k_{\pm} = k \pm \Delta k,$$

where

$$k = \frac{\omega}{c}\sqrt{\varepsilon(\omega)}; \qquad \Delta k = \frac{\omega g(\omega) B_0}{2\sqrt{\varepsilon(\omega)}c}.$$

Eigenmodes are circularly polarized plane waves,

$$\mathbf{E}_{\pm}(z,t) = \hat{\mathbf{e}}_{\pm} E_0 e^{i(k_{\pm} z - \omega t)}.$$

- Magnetic field breaks down the symmetry!

Evolution of a linearly polarized wave

$$\mathbf{E}(0,t) = E_0 \hat{\mathbf{e}}_x e^{-i\omega t},$$

By superposition at any plane $z = const > 0$:

$$\mathbf{E}(z,t) = a_+ \hat{\mathbf{e}}_+ e^{i(k_+ z - \omega t)} + a_- \hat{\mathbf{e}}_- e^{i(k_- z - \omega t)},$$

after some algebra:

$$\mathbf{E}(z,t) = E_0 \hat{\mathbf{e}}_r(z) e^{i(kz - \omega t)},$$

where

$$\hat{\mathbf{e}}_r(z) = \hat{\mathbf{e}}_x \cos \Delta kz - \hat{\mathbf{e}}_y \sin \Delta kz.$$

- Linearly polarized wave with rotating polarization plane

Rotation angle:

$$\Delta\phi = \Delta k L = V B_0 L,$$

where V is the so-called Verdet constant:

$$V = \frac{\omega g(\omega)}{2c\sqrt{\varepsilon(\omega)}}.$$

- Verdet constant gives the rate of rotation
- Application: polarization control.

Linear waves in uniaxial media

Dielectric tensor of a uniaxial crystal with the principal axis along $\mathbf{n}$:

$$\varepsilon_{ij} = \varepsilon_{\|} n_i n_j + \varepsilon_{\perp}(\delta_{ij} - n_i n_j).$$

Illustration

Choose $\mathbf{n} = (0,0,1)$ and $\mathbf{k} = (k_x, 0, k_z)$:

$$\varepsilon = \begin{pmatrix} \varepsilon_{\perp} & 0 & 0 \\ 0 & \varepsilon_{\perp} & 0 \\ 0 & 0 & \varepsilon_{\|} \end{pmatrix}$$

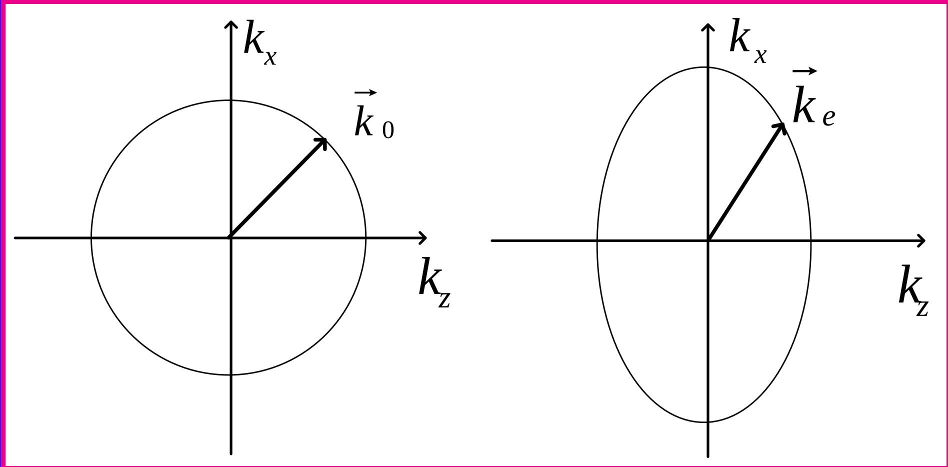
Dispersion relations

- Ordinary waves:

$$k = \frac{\omega}{c}\sqrt{\varepsilon_\perp}.$$

- Extraordinary waves:

$$k = \frac{\omega}{c}\left(\frac{\sin^2\theta}{\varepsilon_\parallel} + \frac{\cos^2\theta}{\varepsilon_\perp}\right)^{-1/2}.$$



$$\frac{\omega^2}{c^2} = \frac{k_x^2}{\varepsilon_\parallel} + \frac{k_z^2}{\varepsilon_\perp}.$$

- Dispersion ellipse

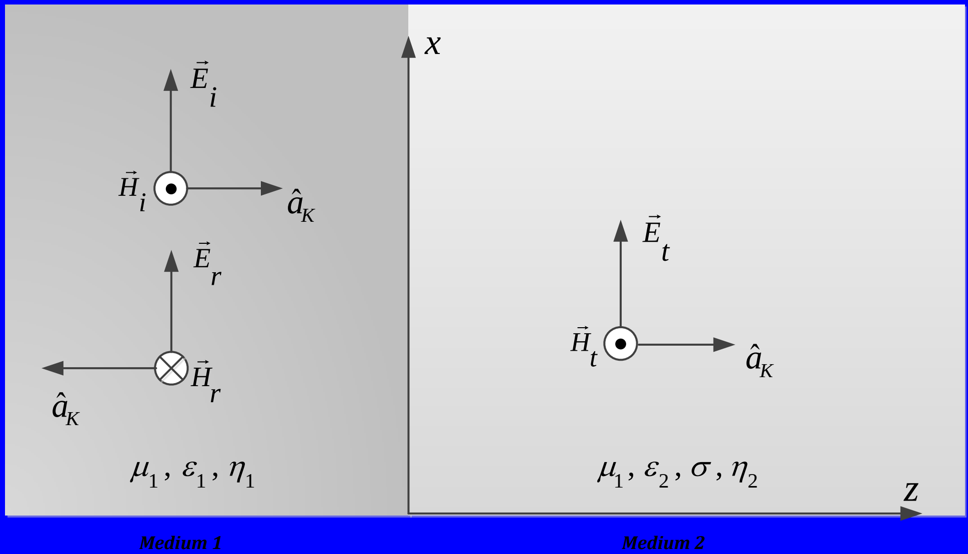
Properties of extraordinary waves:

- Magnitude of the wave vector depends on the propagation direction.
- Propagation direction does not coincide with the energy flow direction – $\mathbf{S}$ need not be parallel to $\mathbf{k}$.

Applications:

- Can be used for phase matching in second-harmonic generation processes.

Reflection of plane waves at normal incidence



Incident wave

$$\mathbf{E}_i(z,t) = \hat{\mathbf{e}}_x E_{0i} e^{i(k_i z - \omega_i t)},$$

and

$$\mathbf{H}_i(z,t) = \hat{\mathbf{e}}_y \frac{E_{0i}}{\eta_i} e^{i(k_i z - \omega_i t)}.$$

Reflected wave

$$\mathbf{E}_r(z,t) = \hat{\mathbf{e}}_x E_{0r} e^{-i(k_r z + \omega_r t)},$$

$$\mathbf{H}_r(z,t) = -\hat{\mathbf{e}}_y \frac{E_{0r}}{\eta_r} e^{-i(k_r z + \omega_r t)}.$$

Transmitted wave

$$\mathbf{E}_t(z,t) = \hat{\mathbf{e}}_x E_{0t} e^{i(\gamma_t z - \omega_t t)},$$

$$\mathbf{H}_t(z,t) = \hat{\mathbf{e}}_y \frac{E_{0t}}{\eta_t} e^{i(\gamma_t z - \omega_t t)}.$$

Boundary conditions

$$\mathbf{E}_{1\tau}|_{z=0} = \mathbf{E}_{2\tau}|_{z=0}, \qquad \mathbf{H}_{1\tau}|_{z=0} = \mathbf{H}_{2\tau}|_{z=0}.$$

imply that

$$e^{i(k_i z - \omega_i t)}|_{z=0} = e^{-i(k_r z + \omega_r t)}|_{z=0} = e^{i(\gamma_t z - \omega_t t)}|_{z=0}.$$

Thus

$$\omega_i = \omega_r = \omega_t = \omega;$$

$$k_i = k_r = k_1 = \frac{\omega}{c}\sqrt{\varepsilon_1},$$

and

$$\gamma_t = \gamma_2 = \beta_2 + i\alpha_2,$$

also

$$\eta_i = \eta_r = \eta_1 = \eta_0/\sqrt{\varepsilon_1},$$

$$\eta_t = \eta_2 = \eta_0/\sqrt{\varepsilon_2}, \qquad \varepsilon_2 = \varepsilon_2' + i\varepsilon_2''.$$

The transmission and reflection amplitudes

$$\boxed{r \equiv \frac{E_{0r}}{E_{0i}} = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2}},$$

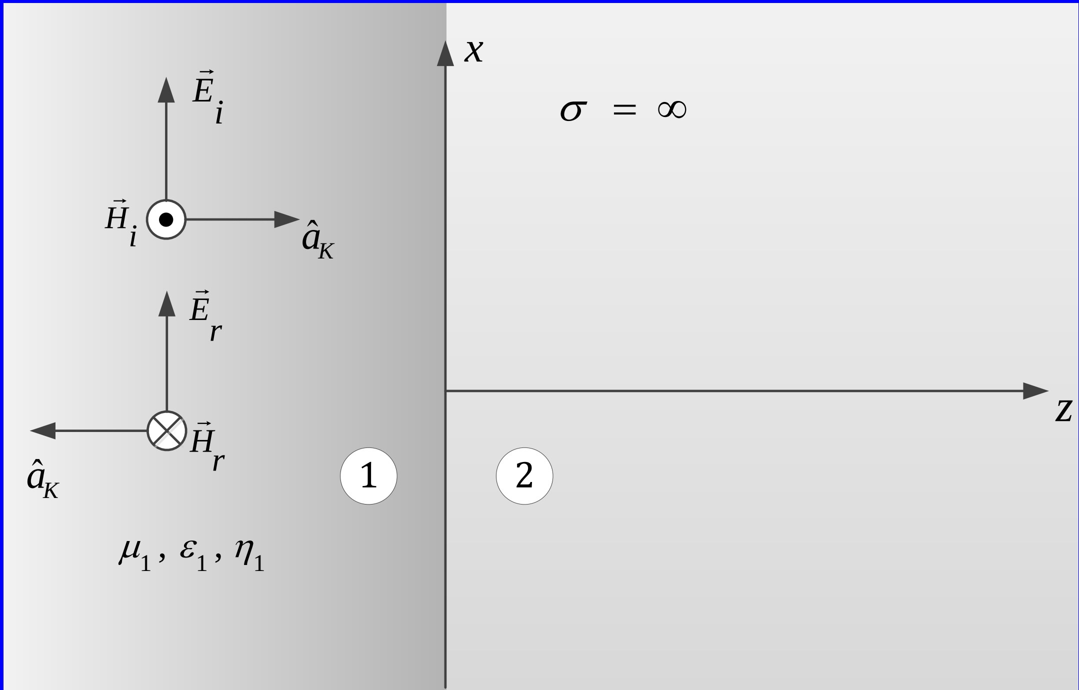
and

$$\boxed{t \equiv \frac{E_{0t}}{E_{0i}} = \frac{2\eta_2}{\eta_1 + \eta_2}}.$$

Energy flux conservation:

$$1 + r = t\,.$$

Perfect conductor, $\sigma \to \infty$



No transmitted wave!

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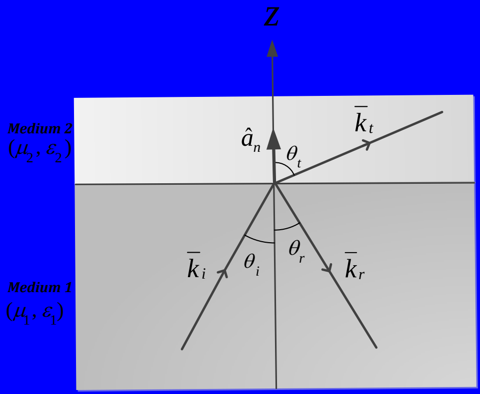
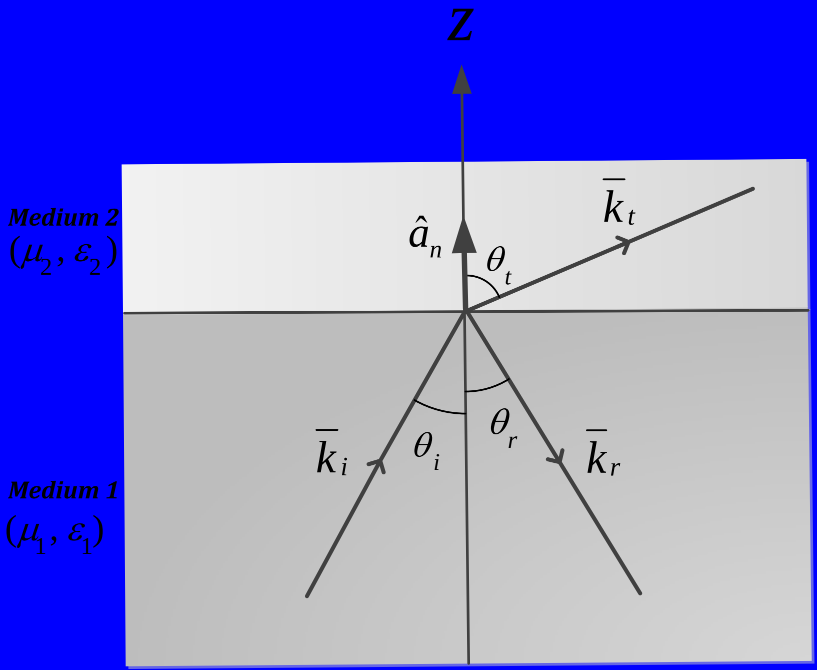
Refraction & reflection at oblique incidence



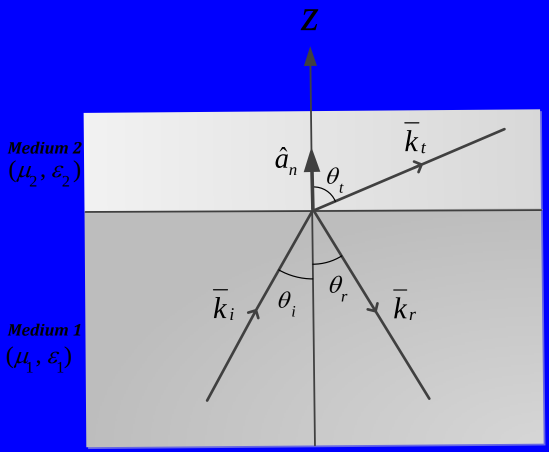
$$\mathbf{k}_i = k_{ix}\hat{\mathbf{e}}_x + k_{iz}\hat{\mathbf{e}}_z,$$

$$\mathbf{k}_r = k_{rx}\hat{\mathbf{e}}_x + k_{rz}\hat{\mathbf{e}}_z,$$

$$\mathbf{k}_t = k_{tx}\hat{\mathbf{e}}_x + k_{tz}\hat{\mathbf{e}}_z,$$



$$e^{i(\mathbf{k}_i\cdot\mathbf{r}-\omega_i t)}\Big|_{z=0} = e^{i(\mathbf{k}_r\cdot\mathbf{r}-\omega_r t)}\Big|_{z=0} = e^{i(\mathbf{k}_t\cdot\mathbf{r}-\omega_t t)}\Big|_{z=0}.$$



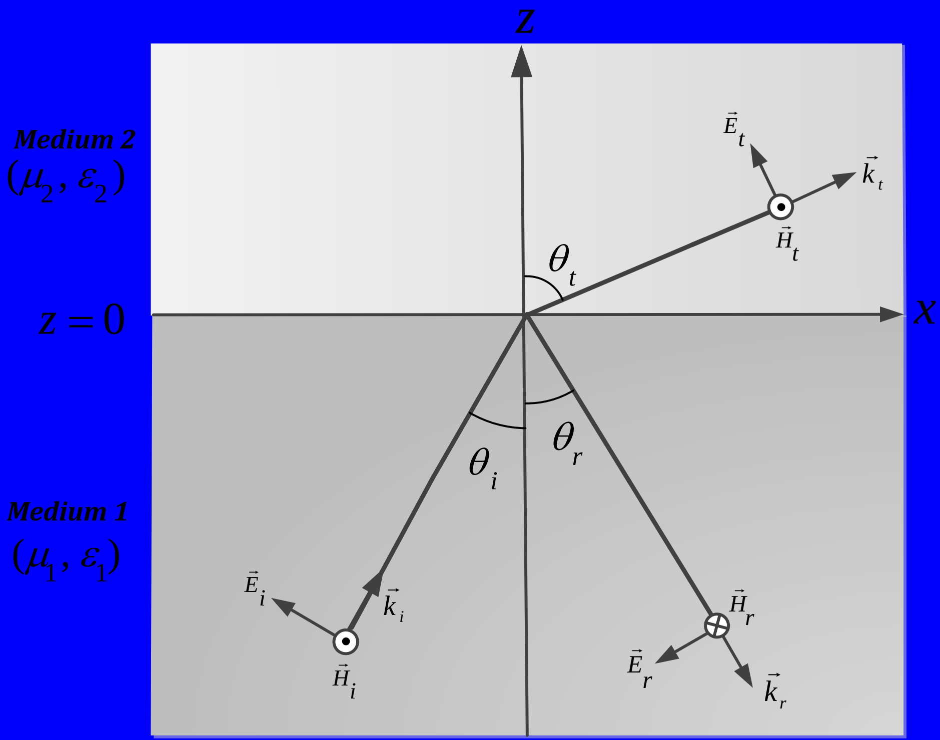
$$k_{ix} = k_{rx} = k_{tx} = k_x$$

$$k_{ix} = k_{rx} \Longrightarrow \theta_i = \theta_r$$

Snell's law

$$\boxed{k_{ix} = k_{tx} \Longrightarrow n_1 \sin\theta_1 = n_2 \sin\theta_2},$$

TM-polarized waves



Incident waves

$$\mathbf{H}_i(\mathbf{r},t) = H_{0i}\hat{\mathbf{e}}_y\, e^{i(\mathbf{k}_i\cdot\mathbf{r}-\omega t)}$$

$$\mathbf{E}_i(\mathbf{r},t) = \eta_1 H_{0i}(k_{iz}\hat{\mathbf{e}}_x - k_x\hat{\mathbf{e}}_z)\, e^{i(\mathbf{k}_i\cdot\mathbf{r}-\omega t)},$$

Reflected waves

$$\mathbf{H}_r(\mathbf{r},t) = -H_{0r}\hat{\mathbf{e}}_y\, e^{i(\mathbf{k}_r\cdot\mathbf{r}-\omega t)}$$

$$\mathbf{E}_r(\mathbf{r},t) = \eta_1 H_{0r}(-k_{1z}\mathbf{e}_x - k_x\mathbf{e}_z)\, e^{i(\mathbf{k}_r\cdot\mathbf{r}-\omega t)},$$

Transmitted waves

$$\mathbf{H}_t(\mathbf{r},t) = H_{0t}\hat{\mathbf{e}}_y\, e^{i(\mathbf{k}_t\cdot\mathbf{r}-\omega t)}$$

$$\mathbf{E}_t(\mathbf{r},t) = \eta_2 H_{0t}(k_{2z}\mathbf{e}_x - k_x\mathbf{e}_z)\, e^{i(\mathbf{k}_t\cdot\mathbf{r}-\omega t)},$$

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TM-reflection and transmission amplitudes:

$$\boxed{r_{TM} = r_{\|} = \frac{E_{0r}}{E_{0i}} = \frac{\varepsilon_2 k_{1z} - \varepsilon_1 k_{2z}}{\varepsilon_2 k_{1z} + \varepsilon_1 k_{2z}}},$$

and

$$\boxed{t_{TM} = t_{\|} = \frac{E_{0t}}{E_{0i}} = \frac{2\varepsilon_2 k_{1z}}{\varepsilon_2 k_{1z} + \varepsilon_1 k_{2z}}\sqrt{\frac{\varepsilon_1}{\varepsilon_2}}}.$$

Transmission and reflection coefficients:

$$R_{TM} \equiv |r_{TM}|^2, \qquad T_{TM} \equiv |t_{TM}|^2.$$

Reflectionless modes

$$r_{TM} = 0 \Longrightarrow, \varepsilon_2 k_{1z} = \varepsilon_1 k_{2z}$$

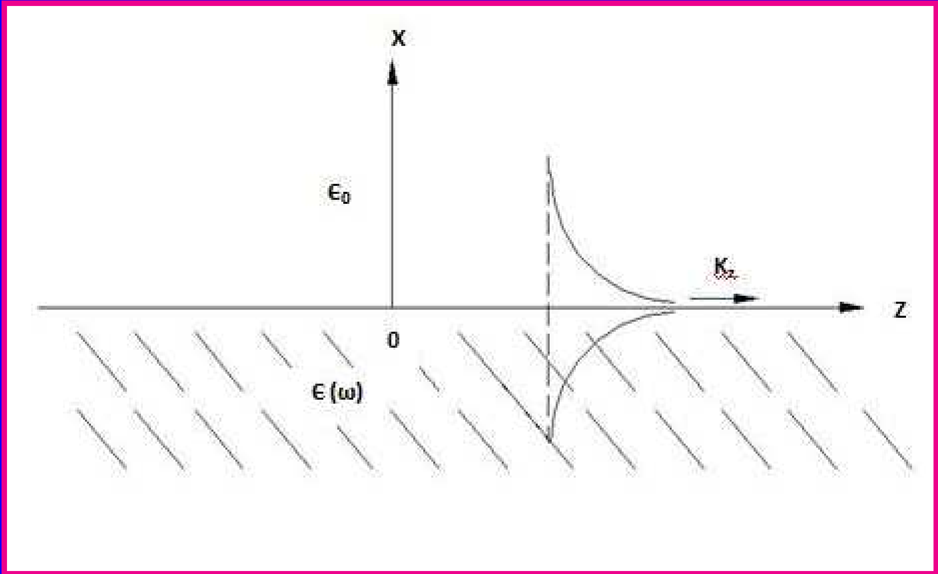
Brewster mode $\Longrightarrow Im\{k_{1z}\} = Im\{k_{2z}\} = 0$

$$\boxed{\tan\theta_B = n_2/n_1};$$

where refractive indices are defined as

$$n_s = \sqrt{\varepsilon_s}; \quad s = 1, 2.$$

Surface plasmon polariton (SPP) modes



$$\varepsilon(\omega) = \begin{cases} \varepsilon_1 > 0, & z > 0, \\ \varepsilon_2(\omega) \le 0, & z < 0. \end{cases}$$

Dispersion relation follows from

$$\varepsilon_2 k_{1z} = \varepsilon_1 k_{2z}$$

and

$$k_{jz} = \sqrt{k_0^2 \varepsilon_j - k_x^2} \quad j = 1,2$$

$$k_0 = \omega / c$$

SPP signature:

$$k_x^2 \geq 0 \quad k_{jz}^2 \leq 0$$

Dispersion relations:

$$k_x = k_0 \sqrt{\frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2}}$$

and

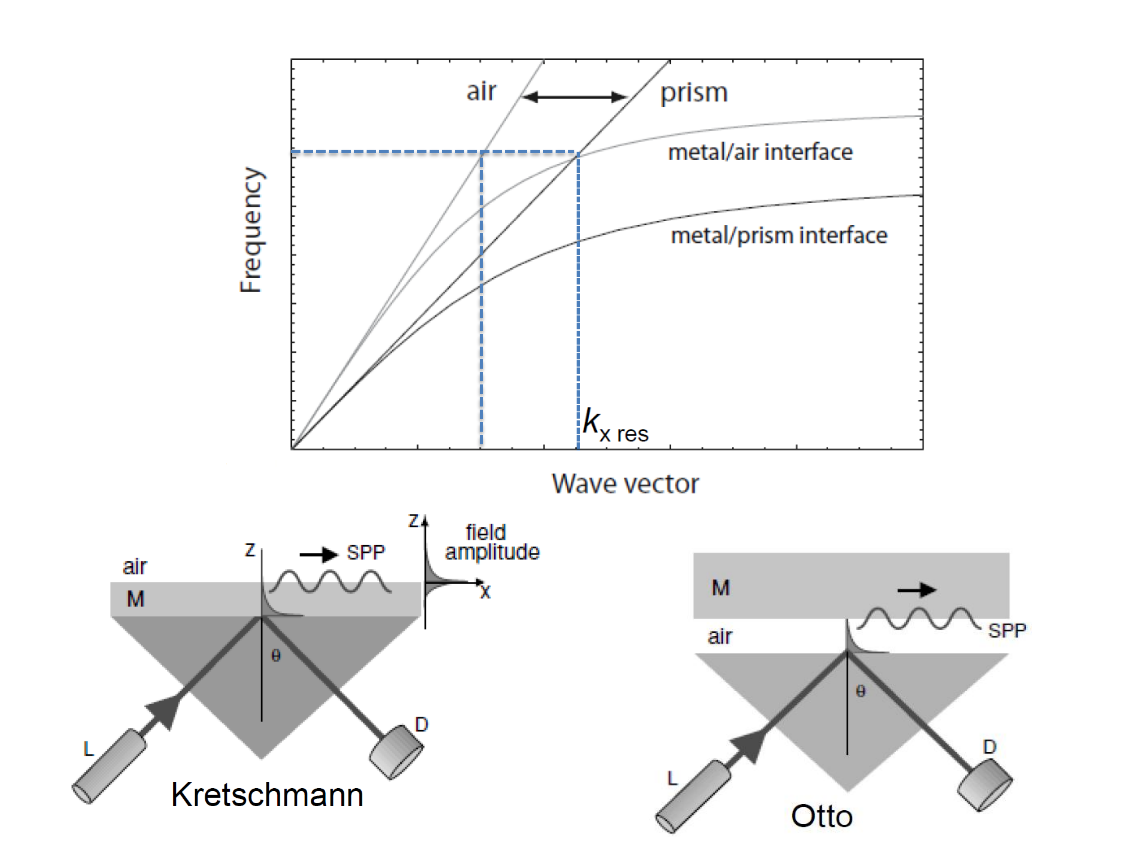
$$k_{jz} = k_0 \sqrt{\frac{\varepsilon_j^2}{\varepsilon_1 + \varepsilon_2}}, \qquad j = 1, 2.$$

SPP conditions:

$$\varepsilon_1 + \varepsilon_2 < 0, \quad \varepsilon_1 \varepsilon_2 < 0.$$

$\varepsilon_1 > 0, \quad \varepsilon_2 < -\varepsilon_1 \Longrightarrow$ metal-dielectric interface

Launching SPPs



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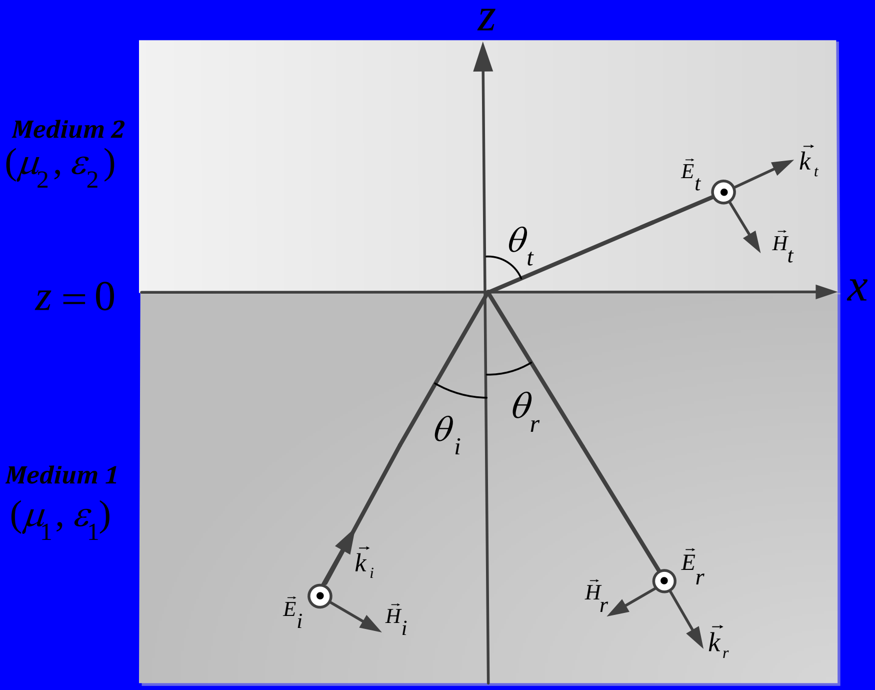
TE-polarization



Incident waves

$$\mathbf{E}_{\mathrm{i}}(\mathbf{r},t) = E_{0i}\hat{\mathbf{e}}_y\, e^{i(\mathbf{k}_i\cdot\mathbf{r}-\omega t)}$$

$$\mathbf{H}_i(\mathbf{r},t) = \frac{E_{0i}}{\eta_1}(-k_{1z}\mathbf{e}_x + k_x\mathbf{e}_z)\, e^{i(\mathbf{k}_i\cdot\mathbf{r}-\omega t)},$$

Reflected waves

$$\mathbf{E}_{\mathrm{r}}(\mathbf{r},t) = E_{0r}\hat{\mathbf{e}}_y\, e^{i(\mathbf{k}_r\cdot\mathbf{r}-\omega t)}$$

$$\mathbf{H}_r(\mathbf{r},t) = \frac{E_{0r}}{\eta_1}(k_{1z}\mathbf{e}_x + k_x\mathbf{e}_z)\, e^{i(\mathbf{k}_r\cdot\mathbf{r}-\omega t)},$$

Transmitted waves

$$\mathbf{E}_{\mathrm{t}}(\mathbf{r},t) = E_{0t}\hat{\mathbf{e}}_y\, e^{i(\mathbf{k}_t\cdot\mathbf{r}-\omega t)}$$

$$\mathbf{H}_t(\mathbf{r},t) = \frac{E_{0t}}{\eta_2}(-k_{2z}\mathbf{e}_x + k_x\mathbf{e}_z)\, e^{i(\mathbf{k}_t\cdot\mathbf{r}-\omega t)}.$$

Transmission and reflection amplitudes:

$$\boxed{r_{TE} = r_{\perp} = \frac{E_{0r}}{E_{0i}} = \frac{k_{1z} - k_{2z}}{k_{1z} + k_{2z}}},$$

and

$$\boxed{t_{TE} = t_{\perp} = \frac{E_{0t}}{E_{0i}} = \frac{2k_{1z}}{k_{1z} + k_{2z}}}.$$

No Brewster angle or SPPs:

$$r_{TE} \neq 0,$$

Total internal reflection

Incidence from more dense medium $n_1 > n_2$

Critical angle:

$$\boxed{\theta_c = \sin^{-1}(n_2/n_1)},$$

Under the condition

$$\theta_1 > \theta_c$$

,

$$k_{2z} = i|k_{2z}| = ik_2\sqrt{\frac{n_1^2}{n_2^2}\sin^2\theta_1 - 1},$$

TM-polarized incident wave

Unimodular reflection amplitude

$$\bar{r}_{TM*} = e^{-2i\phi_{TM*}}$$

where

$$\phi_{TM*} = \tan^{-1}\left(\frac{\varepsilon_1 |k_{2z}|}{\varepsilon_2 k_{1z}}\right)$$

TE-polarized incident wave

Unimodular reflection amplitude

$$\boxed{\bar{r}_{TE*} = e^{-2i\phi_{TE*}}},$$

and

$$\boxed{\phi_{TE*} = \tan^{-1}\left(\frac{|k_{2z}|}{k_{1z}}\right)}$$

Energy flow across the interface?

Time-averaged Poynting vector,

$$\langle \mathbf{S}_t(z) \rangle = \frac{1}{2}\mathrm{Re}[\mathbf{E}_{0t}(z) \times \mathbf{H}^*_{0t}(z)].$$

TM case:

$$\langle \mathbf{S}_t(z) \rangle = \mathbf{e}_x \frac{4\varepsilon_2 k_{1z}^2 k_x}{k_0(\varepsilon_2^2 k_{1z}^2 + \varepsilon_1^2 |k_{2z}|^2)} I_i e^{-2|k_{2z}|z},$$

Propagates along the interface!