Stimulated Raman and Brillouin scattering

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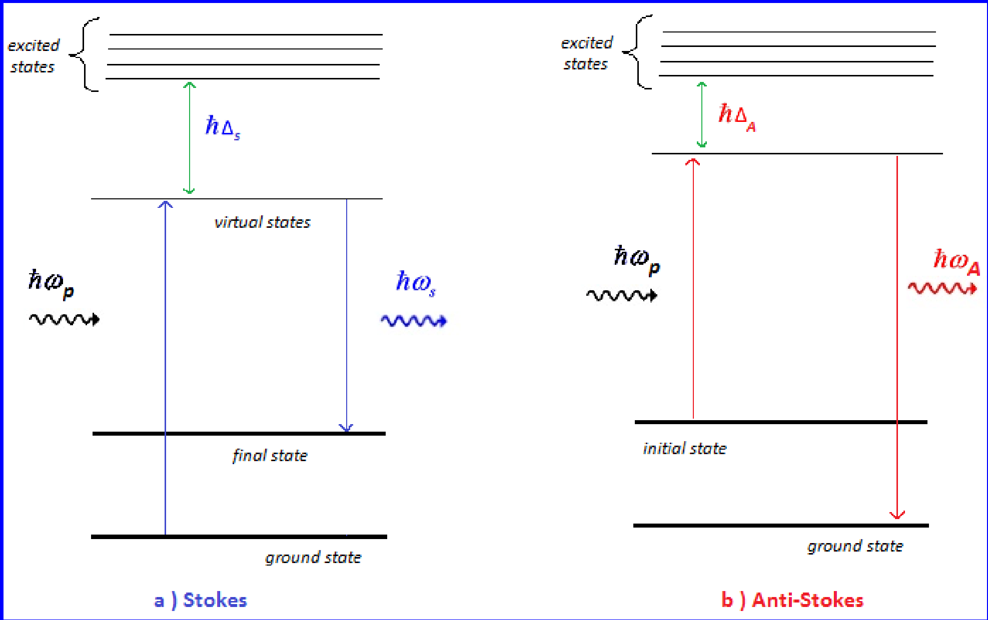
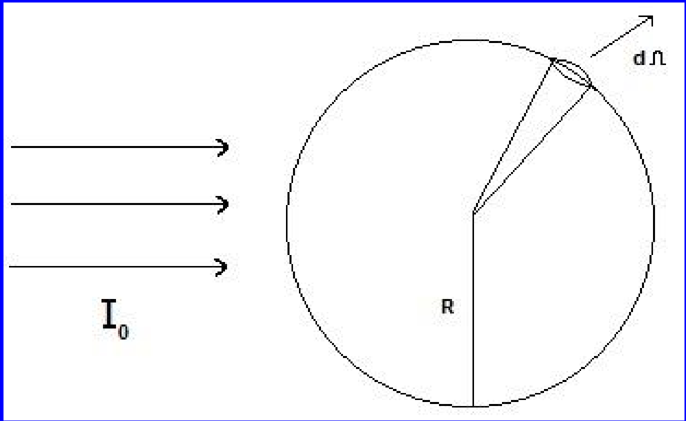
Outline

- Basics of spontaneous Raman scattering
- Stimulated Raman scattering: cw case
- Spontaneous Brillouin scattering
- Stimulated Brillouin scattering (SBS): cw case
- Transient SBS
- SBS gain and threshold

Basics of Raman scattering



$$w_e \propto e^{-(E_f - E_g)/kT} \Longrightarrow \text{anti-Stokes suppressed}$$



Differential scattering cross-section (DSCS)

$$P = \sigma I_0$$

$$\frac{dP}{dS} = \frac{1}{R^2}\left(\frac{dP}{d\Omega}\right) = \frac{I_0}{R^2} \times \underbrace{\left(\frac{d\sigma}{d\Omega}\right)}_{\text{DSCS}}$$

No Stokes mode $\Longrightarrow$ spontaneous scattering

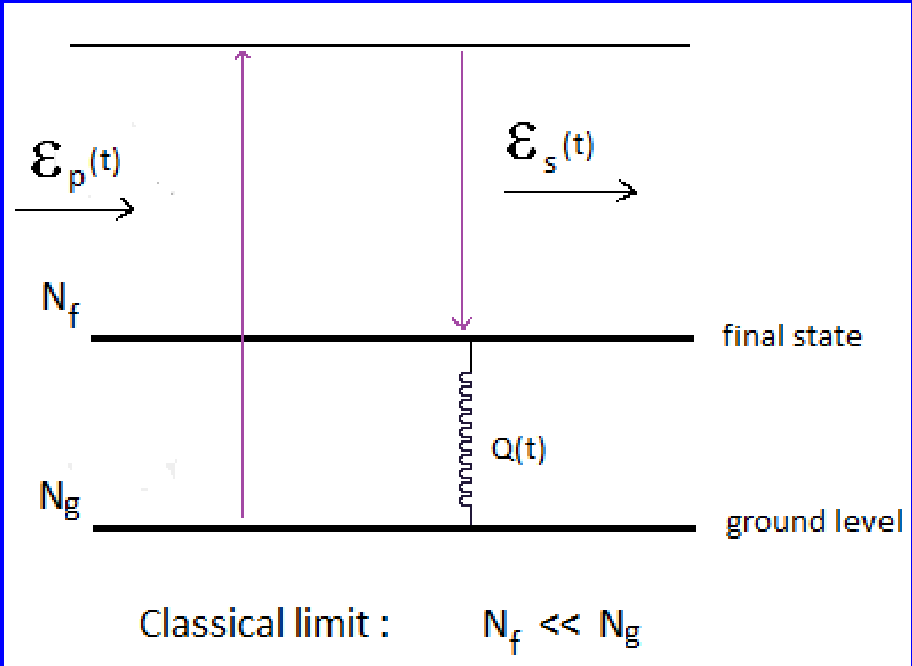
$$P_\Delta = \int_{d\Omega} \frac{dP}{d\Omega} \simeq N I_0 \Delta\Omega \frac{d\sigma}{d\Omega}$$

DSCS tabulated relative to nitrogen,

$$\left.\frac{d\sigma}{d\Omega}\right|_{N_2} = 5.5 \times 10^{-31}\,\mathrm{cm}^2/\mathrm{Sr}/\mathrm{molecule}$$

Conclusion: $P \propto I_0$, weak process!

Classical theory of stimulated Raman scattering (SRS)



Potential energy of a polarized molecule:

$$W = -\frac{1}{2} p E$$

Dipole moment:

$$p = \varepsilon_0 \underbrace{\alpha(\mathcal{Q})}_{\text{polarizability}} E$$

Polarizability:

$$\alpha \simeq \alpha_0 + \left(\frac{d\mathcal{Q}}{d\alpha}\right)_0 \mathcal{Q}$$

Force on a molecule:

$$F_{\mathrm{d}} = -\nabla_{\mathcal{Q}} W$$

Harmonic equation of motion:

$$\partial_t^2 \mathscr{Q} + 2\gamma\partial_t \mathscr{Q} + \omega_0^2 \mathscr{Q} = F_{\mathrm{d}}/m$$

SRS driving electromagnetic field (pump+Stokes):

$$E(t,z) = \frac{1}{2}[\mathscr{E}_p e^{i(k_p z - \omega_p t)} + \mathscr{E}_s e^{i(k_s z - \omega_s t)} + c.\ c.]$$

CW driven solution to molecular oscillator:

$$\mathscr{Q}(t,z) = \frac{1}{2}\mathscr{Q}_\omega e^{i(k_p - k_s)z} e^{-i\omega_\Delta t} + c.\ c$$

$$\mathscr{Q}_\omega = \frac{\varepsilon_0 (d\alpha/d\mathscr{Q})_0 \mathscr{E}_p \mathscr{E}_s^*}{2m(\omega_0^2 - \omega_\Delta^2 - 2i\gamma\omega_\Delta)},$$

$\omega_\Delta = \omega_p - \omega_s$ Raman frequency shift

Induced polarization:

$$P_{NL} = N p_{NL} = \varepsilon_0 N \left(\frac{d\mathscr{Q}}{d\alpha} \right)_0 \mathscr{Q} E$$

Pump and Stokes polarization components:

$$P_{NL} = \tfrac{1}{2}[\mathscr{P}_{NL}(\omega_s)e^{-i\omega_s t} + \mathscr{P}_{NL}(\omega_p)e^{-i\omega_p t} + c.\ c.]$$

Coupled-mode equations:

$$2ik_j \partial_z \mathscr{E}_j = -\mu_0 \omega_j^2 \mathscr{P}_{NL}(\omega_j) e^{-ik_j z}, \qquad j = p, s$$

$$d_z I_p = -g_R I_p I_s$$

$$d_z I_s = g_R I_p I_s$$

Raman gain coefficient:

$$g_R = g_{R0}\mathscr{L}(\omega_\Delta)$$

$$g_{R0} = \frac{\omega_s N (d\alpha/d\mathscr{Q})_0^2}{8m\omega_0\gamma c^2 n_p n_s}$$

Raman gain profile:

$$\mathscr{L}(\omega_\Delta) = \frac{\gamma^2}{[(\omega_\Delta - \omega_0)^2 + \gamma^2]}$$

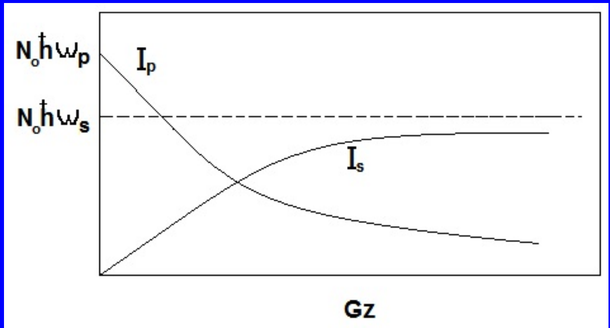
Typical magnitude of center-line gain: $g_{R0} \simeq 1.5$ cm/GW, molecular hydrogen.

$$d_z I_p = -g_R I_p I_s$$

$$d_z I_s = g_R I_p I_s$$

Undepleted pump, $I_p = const$, exponential gain of Stokes mode!

Beyond undepleted pump approximation:



Driving anti-Stokes mode:

$$E(t,z) = \frac{1}{2}[\mathscr{E}_p e^{i(k_p z - \omega_p t)} + \mathscr{E}_s e^{i(k_s z - \omega_s t)} + \mathscr{E}_{as} e^{i(k_{as} z - \omega_{as} t)} + c.$$

Stokes/anti-Stokes shifts:

$$\omega_\Delta = \omega_p - \omega_s = \omega_{as} - \omega_p$$

Material oscillator displacement:

$$\mathscr{Q}(t,z) = \frac{1}{2}\{[\mathscr{Q}_{\omega s} e^{i(k_p - k_s)z} + \mathscr{Q}_{\omega as} e^{i(k_{as} - k_p)z}]e^{-i\omega_\Delta t} + c.c.\}$$

Nonlinear polarization:

$$P_{NL} = \frac{1}{2}[\mathscr{P}_{NL}(\omega_s)e^{-i\omega_s} + \mathscr{P}_{NL}(\omega_{as})e^{-i\omega_{as}} + \mathscr{P}_{NL}(\omega_p)e^{-i\omega}$$

Coupled-mode equations:

$$i\partial_z \mathcal{E}_s = -\xi_s \left(|\mathcal{E}_p|^2 \mathcal{E}_s + \mathcal{E}_p^2 \mathcal{E}_{as}^* e^{i\Delta k z} \right)$$

$$i\partial_z \mathcal{E}_{as} = -\xi_{as} \left(|\mathcal{E}_p|^2 \mathcal{E}_{as} + \mathcal{E}_p^2 \mathcal{E}_s^* e^{i\Delta k z} \right)$$

Phase mismatch:

$$\Delta k = 2k_p - k_s - k_{as}$$

Undepleted pump approximation:

$$I_p = const$$

Undepleted pump + large phase mismatch $\Longrightarrow$ uncoupled Stokes/anti-Stokes modes:

$$d_z I_s = g_s I_p I_s$$

$$d_z I_{as} = -g_{as} I_p I_{as}$$

Exponential gain for Stokes and loss for anti-Stokes!

Spontaneous Brillouin scattering

Light scattering by medium density fluctuations (sound waves):

$$\varepsilon' = \left(\frac{\partial \varepsilon}{\partial \rho}\right)\rho' = \frac{\gamma_e}{\rho}\rho'$$

electrostriction coefficient:

$$\gamma_e \equiv \rho\left(\frac{\partial \varepsilon}{\partial \rho}\right)$$

Induced polarization:

$$P_{NL} = \varepsilon_0 \varepsilon' E$$

Pump field:

$$E = \frac{1}{2}\mathscr{E} e^{i(\mathbf{k}_p\cdot\mathbf{r}-\omega_p t)} + c.c.$$

Sound wave:

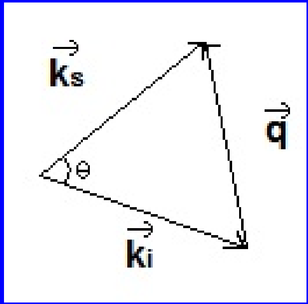
$$\rho' = \frac{1}{2}\tilde{\rho} e^{i(\mathbf{q}\cdot\mathbf{r}-\Omega t)} + c.c.$$

Dispersion laws:

$$\omega_p = k_p c \quad \text{photons}$$

$$\Omega = q c_s \quad \text{phonons}$$

c_s is sound speed.



$$\omega_s = \omega_p - \Omega$$

$$\mathbf{k}_s = \mathbf{k}_p - \mathbf{q}$$

$\Omega \ll \omega_p \Longrightarrow k_p \approx k_s$:

$$\Omega = 2k_p \sin\theta/2$$

Maximum generated phonon energy in the backward direction, $\theta = \pi$!

Brillouin phonon propagation

Sound waves in 1D geometry:

- Mass conservation;
- Momentum conservation (second Newton's law)

Postulating,

$$\partial_t \rho + \partial_z(\rho v) = 0$$

$$\partial_t v + v\partial_z v = -\frac{1}{\rho}\partial_z p_{eff} + (\zeta + \frac{4}{3}\nu)\partial_{zz}^2 v$$

Effective pressure including “negative” electrostriction term:

$$p_{\mathrm{eff}} = p - \gamma_e \frac{\varepsilon_0 E^2}{2}$$

Adiabatic sound speed:

$$c_s = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_s}$$

Sound wave equation with $E = 0$:

$$\partial^2_{tt}\rho' - c_s^2 \partial^2_{zz}\rho' - \underbrace{\Gamma \partial^3_{tzz}\rho'}_{\text{loss}} = 0$$

Sound wave propagation:

$$\rho' \propto e^{i(qz-\Omega t)}$$

Phonon dispersion relation:

$$q \simeq \Omega/c_s + i\alpha/2$$

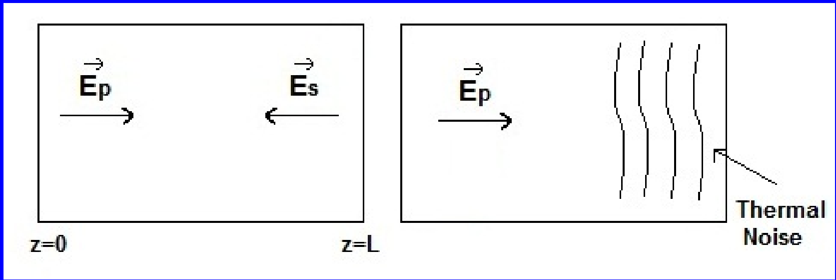
Sound damping rate:

$$\alpha = \Gamma q^2/c_s$$

Estimating, $\Omega/2\pi \simeq 2$ GHz, $\alpha^{-1} \sim 5\mu$m $\Longrightarrow$

- strongly damped ultrasound SBS phonons
- negligible phonon displacements in SBS

Transient SBS (amplification mode)



Driving electromagnetic field:

$$E(t,z) = \frac{1}{2}[\mathscr{E}_p(t)e^{i(k_p z - \omega_p t)} + \mathscr{E}_s(t)e^{i(-k_s z - \omega_s t)} + c.c.]$$

Sound wave:

$$\rho' = \frac{1}{2}\tilde{\rho}e^{i(qz - \Omega t)} + c.c.$$

Kinematics of photon-phonon interaction:

$$q = k_p + k_s \simeq 2k, \qquad \Omega = \omega_p - \omega_s,$$

Density relaxation equation:

$$\partial_t \tilde{\rho} = -(\Gamma_B/2 + i\Delta)\tilde{\rho} + \left(\frac{i\varepsilon_0 \gamma_e k^2}{\Omega_B}\right) \mathscr{E}_p \mathscr{E}_s^*$$

Brillouin frequency detuning,

$$\Delta = \Omega_B - \Omega$$

$$\Omega_B = q c_s \qquad \Gamma_B = \Gamma q^2$$

coupled-wave equations:

$$-\partial_z \mathscr{E}_s + \beta_s \partial_t \mathscr{E}_s = \left(\frac{i\omega\gamma_e}{4cn\rho_0}\right) \mathscr{E}_p \tilde{\rho}^*$$

$$\partial_z \mathscr{E}_p + \beta_p \partial_t \mathscr{E}_p = \left(\frac{i\omega\gamma_e}{4cn\rho_0}\right) \mathscr{E}_s \tilde{\rho}$$

coupled to the SBS phonon relaxation equation:

$$\partial_t \tilde{\rho} = -(\Gamma_B/2 + i\Delta)\tilde{\rho} + \left(\frac{i\varepsilon_0\gamma_e k^2}{\Omega_B}\right) \mathscr{E}_p \mathscr{E}_s^*$$

completely specify transient SBS!

CW limit of SBS:

$$d_z I_s = -g_B I_s I_p,$$

$$d_z I_p = -g_B I_s I_p$$

$$g_B = g_{B0} \frac{\Gamma_B^2/4}{\Gamma_B^2/4 + \Delta^2}$$

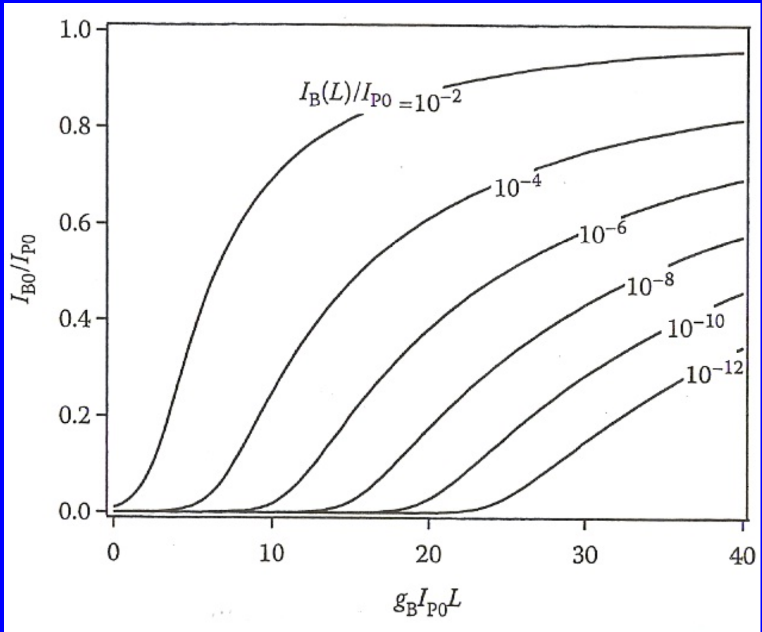
$$g_{B0} = \frac{\omega^2 \gamma_e^2}{n c^3 c_s \rho_0 \Gamma_B}$$

center-line gain estimate $g_{B0} \simeq 0.2$ cm/MW for CS_2, for example.

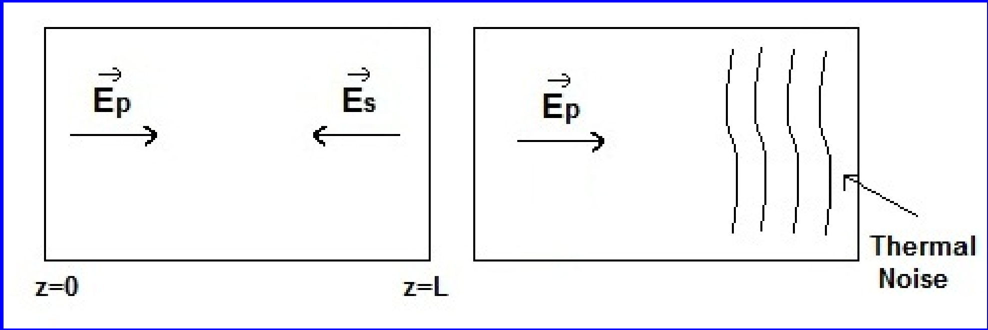
Undepleted pump $I_p = const$:

$$I_s(z) = I_s(L)\exp[g_B I_p (L - z)]$$

Beyond undepleted pump:



SBS threshold



Metric: Reflection coefficient,

$$R = \frac{I_{s0}}{I_{p0}}$$

Stokes intensity at the exit:

$$I_{s0} = I_s(L)e^G$$

Overall gain from noise:

$$G = g_{B0} I_{p0} L$$

Universal gain interval:

$$25 \leq G_{th} \leq 30$$

SBS threshold for cw laser sources

Parameters to consider: source power P, pump beam spot-size w_0,

$$I_{p0} \simeq P/\pi w_0^2$$

Interaction length:

$$L \sim z_R \simeq k_p w_0^2$$

At threshold,

$$G = g_{B0} I_{p0} L = G_{th} \Longrightarrow P_{th} \simeq \frac{G_{th}\lambda_p}{2 g_{B0}}$$

Estimate: $P_{th} \simeq 7$ kW for CS_2.